Investigating the Mathematics of Serial Dilutions

Exploration Two

Interested in the effect of dilution on the concentration of a solution, a chemist compiles the data in table 3 below outlining a different type of dilution. Each step in this dilution uses one ml of the solution prepared in the previous step, which is added to nine ml of water.

Table three

Step	Starting Concentration (%)	Starting volume (ml)	Water added (ml)	Total diluted volume (ml)	Final concentration (%)
1	98	1	9	10	9.8
2	9.8	1	9	10	0.98
3	0.98	1	9	10	0.098
4	0.098	1	9	10	0.0098
5	0.0098	1	9	10	0.00098

1. Use the dilution factor equation below to complete table 4 in your notes for the data in table 3 above. Also determine each concentration relative to the original concentration (9.8)

Dilution factor is defined as: $DF = \frac{finalconcentration}{startingconcentration} x100$

Table four

Step	Dilution Factor (x times)	(final conc./original conc..)
1	______	______
2	______	______
3	______	______
4	______	______
5	______	______

The dilution described in table 3 is called a ***serial dilution***. A *serial dilution* is any dilution where the concentration decreases by the same quantity in each successive step. As you see in table four above, with a dilution factor of 10 the final concentration is 1 / 10 (0.1) the starting concentration and decreases by a factor of 10 with each successive step.

2. Using the information we have learned thus far, what would a graph of final concentration vs. step number for the data in table 3 look like? Make a prediction and sketch a graph.

Use a calculator to construct a scatter graph plotting final concentration vs. step number.

3. What does the shape of graph indicate about the mathematical relationship between final concentration and step number.

4. How was your original prediction validated or refuted?

DIRECTIONS FOR TI-82/83

Y= Clear all equations
STAT EDIT
L1=final concentration (%)
L2=step number
2nd Y=
Plots Off
ENTER
2nd Y=
Plot1
ON
⋰ Xlist=L2 Ylist=L1 Mark: +
ZOOM
ZoomStat

Mathematically it is easier to deal with very large or very small numbers that change by many orders of magnitude by using a logarithm. A logarithm is the exponent (or power) to which a fixed number (base) is raised in order to produce a given number. The *common logarithm* is base 10. Let us investigate the use of logarithms in a serial dilution.

Perform a *logarithmic transformation* of each final concentration by calculating the $\log_{10}$ of each final concentration. (use the data in table 3)

Construct a scatter graph plotting log final concentration vs. step number.

5. Describe the mathematical relationship between log final concentration and step number.

6. Calculate a linear regression for the data and record the equation information.

7. Add the equation line to the graph and sketch the graph.

DIRECTIONS FOR TI-82/83

L3=LOG L1

Y= Clear all equations
STAT EDIT
L2=step number
L3=log final concentration
2nd Y=
Plots Off
ENTER
2nd Y=
Plot1
ON
⋰ Xlist=L2 Ylist=L3 Mark: +
ZOOM
ZoomStat
STAT CALC LinReg(ax+b) L3,L1
ENTER
Y= Y1
VARS Statistics EQ RegEQ
GRAPH

8. What is the significance of the slope of the line (m) for the graph of log concentration vs. step number? The slope of the line should be the log base, which is also the dilution factor.

Consult you teacher for a 500 ppm solution of $KMnO_4$ (potassium permanganate).

9. Perform a serial dilution on your $KMnO_4$ solution. **Using a dilution factor of 5** , continue until the color of the solution is not longer detectable. Complete the following table in your lab report outlining your procedure.

Step	Starting Concentration (ppm)	Starting volume (ml)	Water added (ml)	Total diluted volume (ml)	Final concentration (ppm)	Dilution Factor
1	500	1				5
2						5
3						5
4						5
5						5

10. Once the solution's purple color is no longer detectable with the naked eye is it safe to drink? Explain.

11. Construct an equation that defines the concentration of $KMnO_4$ for each step as the $\log_{10}5$ function of the step number.

12. The Environmental Protection Agency (EPA) currently sets the Maximum Contaminant Level (MCL) for human consumption of $KMnO_4$ at 50 ppm. Use the equation you constructed above to determine the dilution step number when the solution would be safe to drink.

13. How many dilution steps will it take to make the water safe to drink with the DF = 5? How about DF = 10? Show your work in justifying your answer.

14. How many parts per million are present after five steps [dilutions (by 5’s)], and how many parts per ten million is this?

15. After 10 dilution's with the DF = five is the $KMnO_4$ still present in the water? How about 20 steps? 100 steps? Justify your answer.

Extension

Dilutions have special importance for certain 'homeopathic' remedies. See "Homeopathy, cure or quackery", Chem Matters December 1991, pp. 8-13.